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Petri Net Modeling of Routing and Operation Flexibility in Production Systems

G. J. Tsinarakis, N. C. Tsourveloudis, and K. P. Valavanis

***Abstract*— Timed Petri nets are used to model operational and routing flexibility in production systems. A generalized multi-productive machine module is defined, adapted to system features, repeated and connected to compose the TPN models of production systems with different levels of routing and operation flexibility. The models obtained are used to study system parameters under different conditions. Applicability of the method is illustrated through an analytical example.**

I. Introduction

IN the longest period of industrial era, cost was considered to be the most significant factor of manufacturing strategy. This changed in the decade of 1970's, as quality came into highlight as a result of increased competitiveness. Last years, terms such as, flexibility and agility came into focus. The demand of new products has led to shortening of product life cycles, while products manufacturing is made on basis of customer's orders with minimum delivery time. In addition, medium batches of a wide range of products with partially different features are desirable. As a result of the market needs and the available technological tools, manufacturing flexibility has received increasing notice. Flexibility is an attribute of system technology, for coping speedily and efficiently with the variety of environmental needs (e.g. demand), uncertainties (quality changes), disturbances (changes of specifications) and dynamic events allowing companies to become more competitive [1]-[3].

In this work, ordinary timed Petri nets are used to model and study certain aspects of production systems flexibility. A variation of multi-productive machine model introduced in [20] is used to model production systems with different levels of routing and operation flexibility. The models received are used to study system's behavior at different conditions of the manufacturing environment. In the future steps of our research, manufacturing flexibility models obtained will be used for measurement, justification and evaluation of the necessary investments to build an FMS.

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G. J. Tsinarakis is with the Department of Production Engineering and Management, Machine Tools Laboratory, Technical University of Crete, Chania, Greece, GR – 73 100, e-mail: tsinar@dpem.tuc.gr).

N. C. Tsourveloudis is with the Department of Production Engineering and Management, Machine Tools Laboratory, Technical University of Crete, Chania, Greece, GR – 73 100 (Tel.: (+30-28210) 37-285, FAX: (+30-28210) 69-410, e-mail: nikost@dpem.tuc.gr).

K. P. Valavanis is with Department of Computer Science and Engineering, Center for Robot-Assisted Search and Rescue, University of South Florida, 4201 E. Fowler Avenue Tampa, FL 33620-5399, USA. (Corresponding author, e-mail: kvalavan@csee.usf.edu).

II. Production Systems and Flexibility

A. Production systems

Production systems are viewed as a collection of interacting entities for processing initial materials according to specific design specifications, to produce final products with added value. Such systems are composed of human, equipment and supportive entities. Equipment includes processing modules and material handling system that links system components and transports parts between them [8]. In flexible manufacturing systems, machines have advanced capabilities: *i)* choice of machine capable of executing a certain operation from a given set, *ii*) possibility of different operation sequences and *iii)* variety of operations that a machine can perform with short setup [9]. Supportive entities include everything not directly productive.

B. Manufacturing Flexibility

Manufacturing flexibility has been discussed according to different perspectives. This led to a variety of definitions of flexibility types, dimensions and measures. Flexibility has been seen as physical property, attribute of decision-making, economic indicator and strategic tool among others [4]. Literature surveys on flexibility can be found in [5] and [6]. In [5] a scheme is proposed for the classification of the different flexibility aspects found in literature, in categories.

The classification of flexibility types followed here is the one introduced in [9]. The flexibility types identified are: machine, routing, product, operation, process, volume, expansion material handling system, and labor flexibility. In this work routing and operation flexibilities are considered. *Routing Flexibility* is the ability of a production system to manufacture a part using alternative routes in the system. *Operation Flexibility* of a part refers to the ease of changing sequence of the operations required to manufacture the part.

III. Petri Nets

Petri Nets, being both a mathematical and graphical tool, are widely used to study Discrete Event Dynamic Systems. The main reasons that justify that PNs are an ideal tool for modeling production systems with flexibility are: *i)* their ability to capture and represent systems behavior having concurrency, parallelism, resources allocation, mutual exclusion and synchronization *ii)* their ability to model systems hierarchically *iii)* their graphical representation enables easy system visualization *iv)* linkage of desirable system properties to net's structure [15] *v)* their ability to represent resources, activities, constraints and precedence in a single formulation *vi)* possibility to use the PN models for simulation and performance analysis of schedules.

A. Petri Nets formal definition

An ordinary Petri Net (OPN) is formally defined as $\boldsymbol{PN=\{P, T, I, O, m_0\}}$, where $\boldsymbol{P=\{p_1, p_2 \ldots p_{np}\}}$ is a finite set of places, $\boldsymbol{T=\{t_1, t_2 \ldots t_{nt}\}}$ is a finite set of transitions, with $\boldsymbol{P \cap T = \emptyset}$. $\boldsymbol{I:(P\times T)\rightarrow N}$ and $\boldsymbol{O:(P\times T)\rightarrow N}$ are the input, output functions with $\boldsymbol{N}$ the set of non–negative integers. The set of arcs $\boldsymbol{A}$ is partitioned into subsets of standard, test and inhibitor arcs. The special feature of test and inhibitor arcs is that no flow of tokens takes place from them. A test arc of weight r from a place p_i to a transition t_j allows the firing of t_j only if the marking of p_i is greater equal to r, while an inhibitor arc of weight r from a place p_i to a transition t_j allows the firing of t_j only if the marking of p_i is less than r. $\boldsymbol{m_0}$ represents the initial tokens distribution in net's places. A t-timed Petri Net (TPN) results from the corresponding OPN by associating with each transition t_i a firing delay. It is defined as $\boldsymbol{TPN=\{P, T, I, O, m_0, D\}}$ with $\boldsymbol{D}$ representing time delay, a function from the set of non-negative real numbers**.** PN properties (reachability, liveness, safeness, k-boundedness, reversibility and persistency) capture precedence relations and structural interactions between system components [14]-[16].

In the models that follow, places are drawn as circles (◯), transitions as boxes (▮) and tokens are small black dots residing in nets places. Tokens move through arcs according to the respective transitions delays. Standard arcs are drawn as usual (⟶), inhibitor arcs are represented by arcs whose end is marked with a small circle (——○) and test arcs are represented as arcs with dashed line (-·-·-·-→).

B. Use of Petri nets for flexibility study

In [10] PNs model a flexible machine with different part routings according to the machine status. In [11] TPNs are used to introduce a decomposition method to calculate the expected utilization of AGVs and to derive maximum productivity for a given routing. In [12] the routing model of an FMS is constructed using PNs and a heuristic scheduling method based on reachability graph is suggested. In [13] TPNs represent the part routing in FMSs and a beam search based decision method is used for scheduling conflicts resolution. In [21] generic PN models of manufacturing system components are derived and reduction techniques are used to construct functional abstractions of AGV based FMSs. In [22] digraphs and Petri nets are used to model and analyze deadlocks in FMSs. In [17] a deadlock avoidance method applicable to non-sequential resource allocation systems is proposed while in [7] PNs are used to effectively model and analyze conjunctive/disjunctive resource allocation systems.

C. Fundamental module PN model

The fundamental module used to compose the models of production systems with routing and operation flexibility is variation of the multi-productive machine module presented in [20. Here TPNs are used instead of HPNs as in [20], because the systems under study are not mass production systems where large numbers of parts with short processing times are considered. FMSs produce a wide variety of small to medium part batches, so parts number must be discrete to simulate and quantify accurately system's performance.

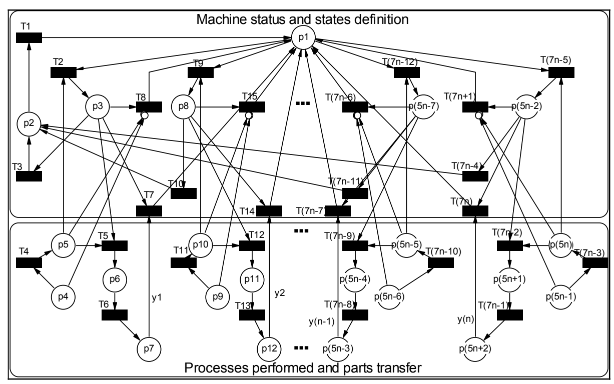

Fig.1. Generalized machine module TPN model.

In the generalized fundamental module, machine $\boldsymbol{M_i}$ receives parts from one of the n upstream buffers (places $p(5n)$, $n=1,\ldots,n_p$) and after processing them sends products to the corresponding downstream buffer (places $p(5n+2)$, $n=1,\ldots,n_p$). In general, machines are not dedicated and perform multiple operation types. In addition, it is possible to execute operations in different sequences unless certain precedence relations restrict this. For each case different input and output buffers are considered, to avoid mixing parts of different types or parts that have received different operations. Also, a different "branch" of the TPN represents each case, since process of different parts in machine have different durations and the time needed to transfer parts from and to buffers also differ. If a machine performs n_1 operations and each operation may be done in n_i different sequences, then the TPN model has $\boldsymbol{n=\sum_{i=1}^{n_1} n_i}$ branches. The meanings of nets nodes are presented in Table I.

TABLE I

FUNDAMENTAL MACHINE MODULE NODES EXPLANATION

Node	*Explanation*
p_1	Machine operational and ready to produce
p_2	Machine out of order
p_{5ni-2}, $n_i=1,\ldots,n$	Machine set-up for n_i type parts process
p_{5ni-1}, $n_i=1,\ldots,n$	n_i initial part buffer
p_{5ni}, $n_i=1,\ldots,n$	n_i type part loaded at machine
p_{5ni+1} $n_i=1,\ldots,n$	n_i type part is processed (process finished)
p_{5ni+2}	n_i final part buffer
t_1	Breakdown repair
t_{7n-5} $n_i=1,\ldots,n$	Set machine for n_i type parts process
t_{7n-4} $n_i=1,\ldots,n$	Machine breakdown while processing n_i type part
t_{7n-3} $n_i=1,\ldots,n$	Supply at machine n_i type part
t_{7n-2} $n_i=1,\ldots,$	Process part
t_{7n-1} $n_i=1,\ldots,n$	Move processed part from machine to output buffer
t_{7n} $n_i=1,\ldots,n$	Full final buffer of n_i type products-change process
t_{7n+1} $n_i=1,\ldots,n$	All n_i type initial parts processed-change process

In the generalized TPN model of Fig. 1 all normal arc weights are 1. The TPN model consists of two parts, one that defines the machine state at each time instant and the second describing processes and parts transports in the system. Change of the part processed in a machine demands change of machine settings. In that time instant, it is possible to select an operation only if initial parts of this type are available. After the repair of a breakdown, the process performed in a machine has to be defined. For this, a test arc connects the place representing parts loaded in the machine with the transition setting machine ready for the respective process. Also, test arcs connect places representing machine setup for parts process with the transitions describing product process. These transitions have another input arc (normal) representing initial part available in the machine. Test arcs of weight y_i connect final buffers with transitions t_{7n} in order to change process performed when a final buffer is full. Inhibitor arcs connect initial buffers and parts loaded at machine with transitions t_{7n} in order to change process when all parts of a type are processed. All the net places except the ones representing buffers have maximum capacities one and all the transitions follow a deterministic or normally distributed time delay.

Modules are derived based on the realistic assumptions: *i)* all buffers are finite and each hosts one part type, *ii)* machines operate at given speeds periodically redefined according to the machine status and the process performed, *iii)* machine breakdowns happen randomly and interrupt immediately net's operation, *iv)* in the PN part describing machine status there is one token during net's operation. This defines the active state of the net at each time instant.

Modules analysis gives similar results with the ones presented in [20]. Considering a TPN model with any finite $\boldsymbol{m_0}$, the following stand: *i)* There exist conflicts in models state definition part, since the machines are non-dedicated. Conflicts are solved according to the system status; *ii)* TPN models are not generally live. No deadlock occurs as long as parts exist in initial buffers; *iii)* Module is *k-bounded* (no self-loops, arc weights being 1 and transitions that generate one token ensure that); i*v)* TPN model is non-persistent and conservative; *v)* Token preservation and machine mutually exclusive states are described by P-invariants; *vi)* TPN model is no repetitive and no consistent. Thus, no T-invariants exist; *vii*) TPN model has $n+1$ P-invariants; one describing alternative machine states and the others parts preservation [20]. *viii)* When connecting two subsequent modules, fusion of places representing buffers happens.

IV. MODELING ROUTING AND OPERATION FLEXIBILITY

The use of TPNs to model routing (RF) and operation flexibility (OF) is pointed out through an analytical example. In this, four TPN models of a production system involving different RF and OF levels are presented.

A. Alternative system TPN models

The system under study consists of five machines and initial parts receive five types of operations to be turned to final products. The precedence relations that define the routing data of the system are presented in Table II.

TABLE II

PRECEDENCE TABLE

Operation	*Predecessors*
1	-
2	-
3	1,2
4	1,3
5	3,2

Scenario 1: A dedicated machine model of the system (transfer line) is constructed (Fig 2.). In this, each machine performs one operation type exclusively and all parts receive operations in the same sequence (**1→2→3→4→5**). This system is completely inflexible, and disturbances cause significant problems in its operation (idleness, reduced productivity etc.). The results of disturbances are rapidly spread in the whole system. In addition, some machines become overloaded and other under-utilized, resulting in increased idleness. This happens as the non-flexible system structure does not allow balancing of the workload [19].

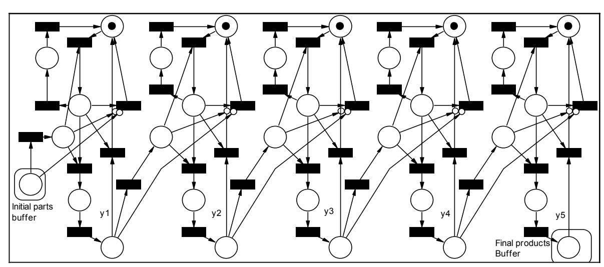

Fig.2. Transfer line TPN model.

Scenario 2: This TPN model (Fig. 3) is characterized by the appearance of operation flexibility (*OF*). Machines are dedicated, but the sequence in which processes are performed is not unique. According to the precedence table, operations 1 and 2 can be performed vice versa as well, while the same stands for operations 4 and 5. In the whole, four alternative operation sequences are possible.

Scenario 3: In this model (Fig. 4) each machine can perform different operation types in different time intervals. In addition, multiple machines can perform an operation with the same or different production features. This system is characterized by the appearance of routing flexibility (RF). Table III shows the operations that each machine can perform. From this it is obvious that machine 3 is dedicated, as is performs exclusively operation 3 and that in this part of the system there is not RF. Machines 2, 4 and 3, 5 respectively perform the same sets of operations.

TABLE III
OPERATION TYPES PERFORMED BY EACH MACHINE

Operation	*Machine*
1	1, 5
2	2, 4
3	3
4	4, 2
5	5, 1

In this case, sixteen alternative operation sequences can be used for items production. The alternative routes are represented with pairs (operation, machine that performs it, e.g. $\mathbf{(1,1) \to (2,2) \to (3,3) \to (4,4) \to (5,5)}$).

Scenario 4: In the final TPN model considered (Fig. 5), RF and OF are combined in a system. Machines are not dedicated and the described precedence relations are valid. In this model sixty-four alternative operation sequences (4*16=combinations of the previous models) are possible. For the alternative sequences representation 3-character variables having the form (operation sequence, operation number, machine) are used. Table IV summarizes the nodes complexity and the alternative paths of the TPN models.

TABLE IV
ALTERNATIVE TEST SCENARIO FEATURES

Test Scenario	*Places*	*Transitions*	*Paths*
1	31	40	1
2	45	68	4
3	43	68	16
4	69	124	64

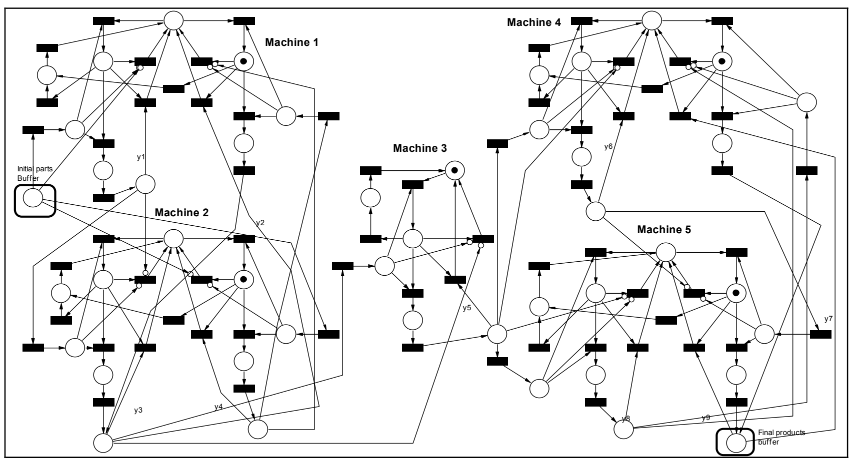

Fig.3. TPN model of a production system with operation flexibility.

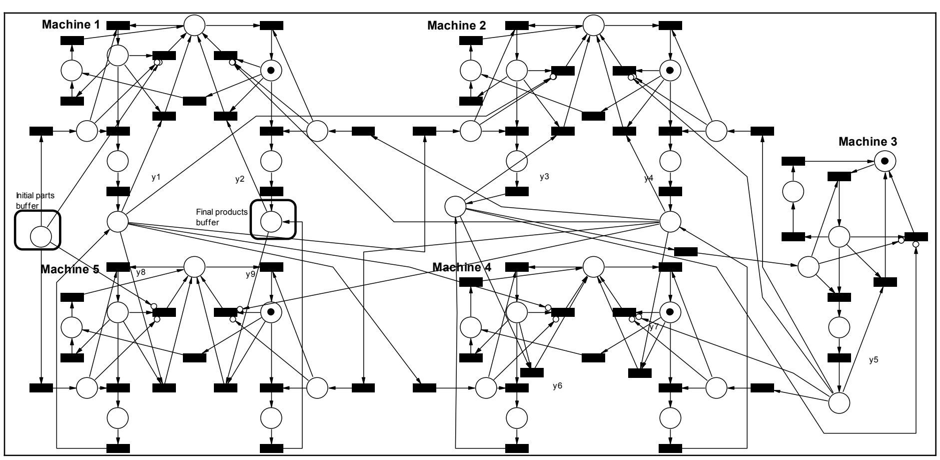

Fig.4. TPN model of a production system with routing flexibility.

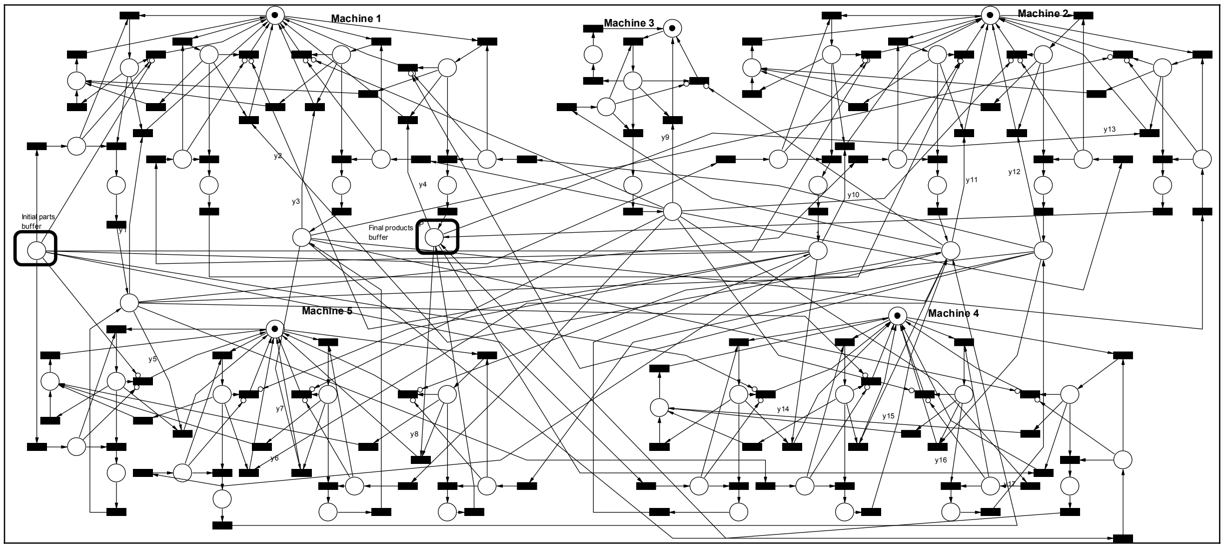

Fig.5. TPN model of a production system with routing and operation flexibility.

A. *Performance evaluation of alternative models*

Simulations of the TPN models are performed to calculate a number of parameters and how these change according to change of certain net features. Simulations are performed using Visual Object Net [18].

In the simulations the delays in all the respective net transitions are considered the same (for example, time to perform all operations is the same in all machines of all scenarios and all machine set-ups have the same duration) so that results with practical value are received.

In the first experiment the effect of different flexibility types in net's operation is studied. For this, the performance of a task has the same duration in all four models. The target is the production of 30 items, all initially found in raw materials buffer, in the minimum time. Durations of tasks performed in the system are presented in Table V.

TABLE V
DURATIONS OF PERFORMED TASKS

Task	*Delay (time units)*
Operation (process) i, $i=1,5$	2.5
All machine set-ups	0.5
Machine i breakdown repair, $i=1,5$	3.5
Parts transfer to machine i, $i=1,5$	1.5
Transfer of part from machine to buffer after operation i, $i=1,5$	1

Four different cases for each TPN model are considered. Initially, all internal buffer capacities (*BC*) of the system have capacity of 3 parts, while in the second case *BC* is increased to 4. The breakdowns appearance (*BT*) follows a normal distribution in (0, 20). This is represented in Table VI as $BT \in (0, 20)$. In the third case *BC* is 3 and *BT* is a random number normally distributed between 0 and 25. In the final case *BC* of 4 parts and *BT* in *(0, 25)* are considered concurrently. In all cases sets of 10 simulations are performed and the mean parameter values are calculated.

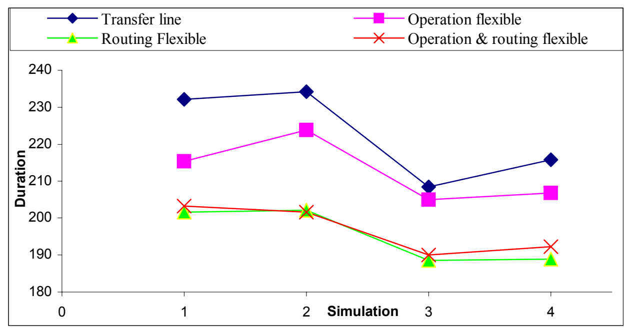

Fig.6. Mean simulation times for the alternative production scenarios.

Calculated simulation durations that express production times, are illustrated in Fig. 6. From these it is obvious that minimum production time is derived when reliability is increased. In addition, routing flexible and operation and routing flexible systems need shorter time intervals to end their process while transfer line needs the longest one.

In order to study the effect of machine reliability, another experiment is performed. For the routing flexible system, for constant capacities equal to 4 and for different machine reliabilities, simulations for the production of 30 items are performed. Results are presented in Table VI. It is obvious that production time reduces as reliability increases, until a limit is reached where breakdown appearances are rare and their effect in PNs operation is not significant.

TABLE VI
SIMULATION TIMES FOR THE DIFFERENT MACHINE RELIABILITIES

$BT \in (0,15)$	$BT \in (0,20)$	$BT \in (0,25)$	$BT \in (0,30)$	$BT \in (0,35)$	$BT \in (0,40)$
225.06	202.11	188.85	176.08	172.6	172.58

Then, the behavior of the routing flexible system is studied increasing internal buffers capacities for constant machine reliability $(BT \in (0, 25))$. Results are presented in Table VII. For $BC = 2\text{-}4$ simulation time is almost constant while for $BC = 5$ a significant reduction takes place. For *BC*

=*6, 7* simulation time increases, probably because of increased machine idleness due to full buffers. Finally for *BC = 10* minimum simulation time is obtained. For buffer capacities bigger than 10 the simulation time is constant.

TABLE VII
SIMULATION TIMES FOR THE DIFFERENT MACHINE RELIABILITIES

BC=2	*BC=3*	*BC=4*	*BC=5*	*BC=6*	*BC=7*	*BC=10*	*BC=15*
200.03	201.56	202.11	191.91	196.32	197.24	189.66	189.53

The final experiment concerns change in the number of processed parts. Simulations are performed for all the four scenarios concerning the production of 10, 30, 60, 100 and 150 items. Results are presented in Table VIII. From these we can see that transfer line in almost all cases needs the longest time period to produce the items. In addition, operation flexible system needs also long time compared to the other systems while routing flexible and operation and routing flexible systems need the shortest time periods

TABLE VIII
SIMULATION TIME FOR PRODUCTION OF DIFFERENT NUMBERS OF ITEMS

Scenario/ Number of parts	*10*	*30*	*60*	*100*	*150*
Transfer Line	89.1	208.43	395.75	617.25	919.68
Operation Flexible	93.55	204.95	376.96	600.27	883.31
Routing Flexible	77.49	188.56	350.76	584.9	870.73
Operation & Routing Flexible	85.95	190.01	356.86	570.18	859.34

B. *Discussion*

As pointed out by the simulation results, flexibility in general results in non-decreasing performance of the system productivity. However, to increase flexibility it is necessary to increase the cost for the investment, as except the usual costs, there are increased costs for parts transfer, backup machines, tool costs and increased need for storage spaces. The trade-off between flexibility and cost must be taken into account during system design to obtain the necessary flexibility and maximum efficiency in the derived system.

In the next steps of our research the influence of other factors as assigning different priorities in the performed operations, control strategies followed and other flexibility types will be studied based on the fundamental model.

C. *Conclusions*

A TPN-based methodology for modeling of operational and routing flexibility in production systems is proposed. The proposed method enables the study of systems with different flexibility levels and types. TPNs unlike other graph-based modelling methods can represent at the same time precedence relations, feasible operation sequences and the machines available to perform each operation. In addition the models obtained can be used to simulate different production scenarios and obtain useful conclusions concerning the optimal design and operation of the system.

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