Fuzzy Assessment of Machine Flexibility

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***Abstract*— Manufacturing flexibility is a difficult and multifaceted concept that because of its inherent complexity and fuzziness is amenable to an artificial intelligence treatment. Fuzzy logic offers a suitable framework for measuring flexibility in its various aspects. This paper deals with the measurement of machine flexibility. When data are precise, this is done via a simple analytical formula. But if such data, and hence knowledge, are not precise, fuzzy-logic modeling should be employed by transforming the human expertise into IF-THEN rules and membership functions. An implementation of the interval-valued fuzzy-set approach, together with a max-min schema, provides the approximate inference mechanism for the computation of machine flexibility. This approach has the advantage of revealing second-order semantic uncertainty with the associated nonspecificity measure. The models are illustrated with a number of examples.**

***Index Terms*— Approximate reasoning, flexibility measures, fuzzy modeling, linguistic rules, machine flexibility.**

I. Introduction

TRADITIONAL manufacturing has relied on dedicated mass-production systems to achieve high production volumes at low cost. As living standards improve and the demand for new consumer goods rises, manufacturing flexibility gains great importance as a strategic weapon against rapidly changing markets. Flexibility, however, cannot be properly incorporated in the decision-making process if it is not well defined and measured in a quantitative fashion. Today, manufacturing flexibility remains an elusive concept because of its inherent complexity and generality, in spite of a large body of research that has been published. There exist more than 50 definitions of [1] and six different approaches to obtaining a quantitative flexibility measure [2]. Flexibility in its most rudimentary essence is the ability of a manufacturing system to respond to changes and uncertainties associated with the production process [3]–[5]. A comprehensive classification of eight flexibility types was proposed in [6]. Resource and system flexibilities were examined in [7], whereas global measures for flexible manufacturing systems (FMS's) were defined in [8]. Routing flexibility based on information theoretic concepts was examined in [9] and [10]. Flexibility measures for one machine, a group of machines, and a whole industry were presented in [11], involving appropriate weights and machine efficiencies in carrying out sets of tasks. In [12], the period needed by a system to recover after a change was used as the central flexibility measure, whereas a stochastic dynamic programming model for its assessment was presented in [13]. Artificial intelligence (AI) methods seem appropriate in most practical situations where numerical data are not readily available and linguistic variables are more amenable in handling imprecise knowledge [14]. The flexibility of competing systems can be ranked appropriately using an algorithmic approach [15] or a decision support system [16] based on performance and economic criteria. Last, integer programming methods have been proposed in [17], and a graphical representation method of production processes in [18].

Manufacturing flexibility is associated with uncertainty in all levels of a firm's operation, such as variation in the demand and characteristics of a product or unanticipated interruptions of the production process because of machine failures. In addition, human operators or managers use imprecise concepts and vague meanings when they attempt to define or measure flexibility. Fuzzy-set theory [19], [20], and especially fuzzy logic, constitutes a natural framework for the representation and manipulation of uncertainty.

Indeed, fuzzy-set theory is an algebra of imprecise propositions and gradual statements such as "machine A is more flexible than machine B because it is more versatile." In previous treatments, uncertainty is handled by probability theory under the assumption that probabilities can be obtained precisely. Mandelbaum and Buzacott [21], examining the meaning and use of flexibility in decision-making processes, admit that for real-world problems with increased complexity, the existing modeling methods are inadequate to represent reality. For context-dependent situations where conceptual imprecision exists, however, as in the description of machine flexibility itself, fuzzy sets and logic appear to be more appropriate for the definition and analysis of the problem. The use of fuzzy sets in assessing flexibility, to our knowledge, has not been introduced elsewhere in the literature. We hope that this approach will prove fruitful and certainly provide a different, and perhaps more natural, vantage point to assess flexibility.

In this paper, we adopt the AI approach and construct a rule-based system to handle imprecise data about a production system. We are dealing with the measurement of machine flexibility, and develop a model whose variables cannot be computed precisely, namely, machine versatility, adjustability, and setup times. Although certain factors that affect flexibility can be quantified by managers or flexibility experts, their

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functional relationship to flexibility is not so clear. As an alternative, decision makers often use natural language ratings and qualitative assessments to measure various flexibilities. Here, we use fuzzy modeling implication methods in a flexibility-measurement methodology that is easy for a manager to interpret and use, as we shall see.

First, we provide a mathematical expression for the computation of comparative machine flexibility when precise data are available about setup times, number of operations, and range of adjustments for each machine. When these quantities are not known precisely, we introduce linguistic modeling, which is performed by extracting knowledge from human experts and representing their perceptions via appropriate linguistic IF-THEN rules. The linguistic variables involved in these rules are production parameters whose values are fuzzy sets. The attraction of these models is that they imitate natural language expressions such as, "If the importance is small then the priority is low," and thus they capture vagueness, which is an inherent property of human communication. Approximate reasoning, together with a certain class of fuzzy-logic operators, is employed in the last part of the paper to compute machine flexibility. A number of examples illustrate these ideas.

II. Machine-Level Flexibility

It is generally accepted that flexibility of manufacturing systems is a multidimensional but also vague notion, which for reasons of simplicity has been broken down into several distinct types. In the definition of flexibility, a machine is the basic hierarchical element of the production process. These definitions can be extended to more general hierarchies such as groups of machines or a whole industry. All manufacturing systems are flexible to some extent, but flexibility here refers to production systems consisting of a set of numerically controlled machines connected by a transportation system and controlled by a central computer. Machines are equipped with exchange mechanisms for tools and workpieces, which enable them to perform several operations in a given configuration with short load, unload, and tool exchange times. Machine flexibility (MF) is the simplest type of the concept, which constitutes a necessary building block for the determination of more complex notions such as product, process, and operation flexibilities [22]. MF in itself, although basic, is quite difficult to compute because of its nature. For example, it is rather well understood how operational characteristics (setup, product variety, etc.) individually influence MF, but the overall effect cannot be satisfactorily provided by analytical expressions.

The observable components we use in our study for the measurement of MF are:

1) the setup time required for various preparations (tool preparations, part positioning and release time, CNC changeover time, software changes, etc.), which represents the ability of a machine to absorb changes in production efficiently;
2) the variety of operations a machine can perform, which will be called machine versatility[1];
3) the range of adjustments, which is defined as the size of the working space and is related to the part sizes a machine can produce.

[1] For a detailed discussion about versatility as a flexibility factor, see [23].

These parameters, called primary data, express changeover speed, versatility, and adjustability, which are among the most common and important aspects of MF. To evaluate each parameter, we need information about 1) batch sizes, 2) variety of products, and 3) variety of product sizes. We call these parameters secondary data and use them to determine weights. The general rules for assigning weights of importance to the primary data are:

- changeover speed is *very important* when batch sizes are *small*;
- versatility is *very important* when product variety is *high*;
- adjustability is *very important* when products have different sizes.

A. A Comparative Measure

Now we provide a comparative measure of MF when crisp numerical data are available. Comparative measures are useful to management in choosing the system that most appropriately fits certain requirements, especially FMS's. Measures of comparative MF have been reported in [15] and [24] using only setup times. Our measure ranks alternative machines or machining centers according to their flexibility by taking into account the major components of MF as well as the management's belief about them. In this sense, the proposed measure is user oriented. Specifically, let $M = \{1, \cdots, m\}$ be the set of competing machines. Then the flexibility measure MF_i of machine i is given by

$$\mathrm{MF}_i = W_s \frac{\min_j[s_j]}{s_i} + W_v \frac{v_i}{\max_j[v_j]} + W_r \frac{r_i}{\max_j[r_j]}, \quad i, j \in M \tag{1}$$

where

s_i	setup time of machine i;
v_i	number of operations machine i can perform;
r_i	range of adjustments of machine i;
W_s, W_v, W_r	weights of importance for s_i, v_i, and r_i;

and $W_s + W_v + W_r = 1$.

In (1), MF is the weighted sum of the relative values of s_i, v_i, and r_i compared to the best machine values. For the case of s_i, the best value corresponds to the machine with the smallest setup time. According to the information above, if a machine happens to outrank the others in the sense of possessing the smallest s and the largest v and r, then its relative flexibility equals one.

Note that the weights are subjectively chosen according to the particular type of installation studied. We illustrate (1) through an example.

Suppose a set of five machining centers (mc's) are to be compared with the following data:

	s	v	r
mc_1	5.53	9	107.3
mc_2	7.21	17	98.6
mc_3	6.32	12	103.7
mc_4	10.01	20	120
mc_5	7.82	15	115.

Machining center mc_1 has the smallest setup time, while mc_4 takes the larger value in all three parameters. Furthermore, suppose that the weights for each parameter are $W_s = 0.3, W_v = 0.5$, and $W_r = 0.2$. From (1), we have

$$\mathrm{MF}_1 = 0.3\frac{5.53}{5.53} + 0.5\frac{9}{20} + 0.2\frac{107.3}{120} = 0.7038.$$

Similarly, it is found that $\mathrm{MF}_2 = 0.8193, \mathrm{MF}_3 = 0.7353, \mathrm{MF}_4 = 0.866$, and $\mathrm{MF}_5 = 0.778$, which shows that mc_4 is the most flexible among all candidate mc's. Let us suppose now that a new machining center mc_6 of the same class is under consideration, with values $s_6 = 6.5, v_6 = 15$, and $r_6 = 108$. Directly from (1), we compute $\mathrm{MF}_6 = 0.8102$, and since $s_6 > s_1 = \min[s_1, \cdots, s_6], v_6 < v_4 = \max[v_1, \cdots, v_6]$, and $r_6 < r_4 = \max[r_1, \cdots, r_6]$, the previously computed flexibilities remain unaltered. Obviously, if the new machine has a better score in any of the three parameters, e.g., $s_6 > s_1$ or $v_6 > v_4$ or $r_6 > r_4$, then the MF should be recalculated according to the new data. This is an expected and desirable property of (1).

It should be stressed again that (1) provides a measurement of relative flexibility that is conceptually simple and aims to help managers in choosing among several alternatives by taking flexibility as a criterion. Indeed, in practice, we mostly need a measurement tool to certify whether a machine A is more flexible than a machine B.

III. A Fuzzy Perspective

Precise data and weighting factors usually are not available in practical situations. The factors that affect MF are not independent. Machine versatility, for example, has a large impact on setup time. A multipurpose machine, which is capable of performing many different operations, minimizes the setup time needed for the production of a certain class of parts. An additional complication is that precise data and weights concerning these factors usually are not available in practical situations. Setup time, for instance, is allocated to part and tool positioning and release, software changes, etc., which cannot be measured easily. Bearing this in mind, managers prefer linguistic to numerical values in measuring factors affecting machine flexibility. In many cases, researchers have utilized natural language expressions such as *high, low,* or *fair* in their attempt to evaluate flexibility. Generally, the representation of managerial knowledge by linguistic rules performs better when there are no units of measurement for the attributes of the system and no quantitative criteria for the values of such attributes [25]. Fuzzy logic offers a systematic base in dealing with such cases. Motivated by this fact, we introduce a rule-based measurement scheme in which no analytic or numerical expressions of flexibility parameters are required. Rules contain already known facts but in compact form, such as, "If setup time is *low*, then machine flexibility is *high*," in which the linguistic values (*low* and *high* here) are represented by appropriate fuzzy sets. The value of flexibility is the result of a fuzzy or approximate reasoning procedure. We now provide some background material of fuzzy-set theory.

Let X be a collection of objects, called the universal set, whose elements are denoted by x. Then a fuzzy set A in X is defined as the set of ordered pairs

$$A = \{(\mu_A(x)/x) | \mu_A(x) \in [0,1], \qquad x \in X\}$$

where $r_A(x)$ is the membership function of $x \in X$ in A. The membership function denotes the degree to which x belongs in A. The closer the value of $r_A(x)$ is to one, the more x belongs to A. Membership functions are not unique, as different people might define various membership functions for the same fuzzy set. The *union* of two fuzzy sets A and B is also a fuzzy set $A \cup B$ such that

$$\mu_{A\cup B}(x) = \max[\mu_A(x), \mu_B(x)], \qquad \text{for every } x \in X.$$

The *intersection* $A \cap B$ is

$$\mu_{A\cap B}(x) = \min[\mu_A(x), \mu_B(x)], \qquad \text{for every } x \in X.$$

The standard *complement* A^c of a fuzzy set A has the membership function

$$\mu_{A^c}(x) = 1 - \mu_A(x), \qquad \text{for every } x \in X.$$

This "max-min-standard complement" is a simple extension of the classical set theory operations and is known as the Zadeh's De Morgan Triple. Other extensions are possible as well [26], e.g., $\mu_{A\cap B}(x) = \mu_A(x) \bullet \mu_B(x)$ or $\mu_{A\cup B}(x) = \min[1, \mu_A(x) + \mu_B(x)]$.

A fuzzy *conditional statement* or a fuzzy if/then rule is an expression of the type "if X is A then Y is B," denoted $A \to B$, where A and B are values of the linguistic variables X and Y. A linguistic variable is mainly characterized by:

a) its linguistic values, e.g., A, B;

b) the physical or abstract concept domain over which the variables, e.g., X, Y, take their quantitative values;

c) the membership function of the linguistic values, e.g., $\mu_A(x), \mu_B(y)$.

For example, *versatility,* which is an abstract concept, can be regarded as a linguistic variable of MF taking linguistic values such as *low, about low, average, high,* and so on. The physical domain of *versatility* is the set of different operations v that the machine can perform over a set of products, which, for example, for most mc's is represented by the interval [0, 35].

The meaning of the fuzzy conditional $(A \to B)$ is given by a fuzzy relation

$$\mu_R(x,y) = \mu_A(x) \otimes \mu_B(y), \qquad x \in X, y \in Y$$

where $\otimes$ represents any fuzzy implication. Fuzzy relations play a major role in fuzzy or approximate reasoning. The most frequently used inference method, the *Compositional*

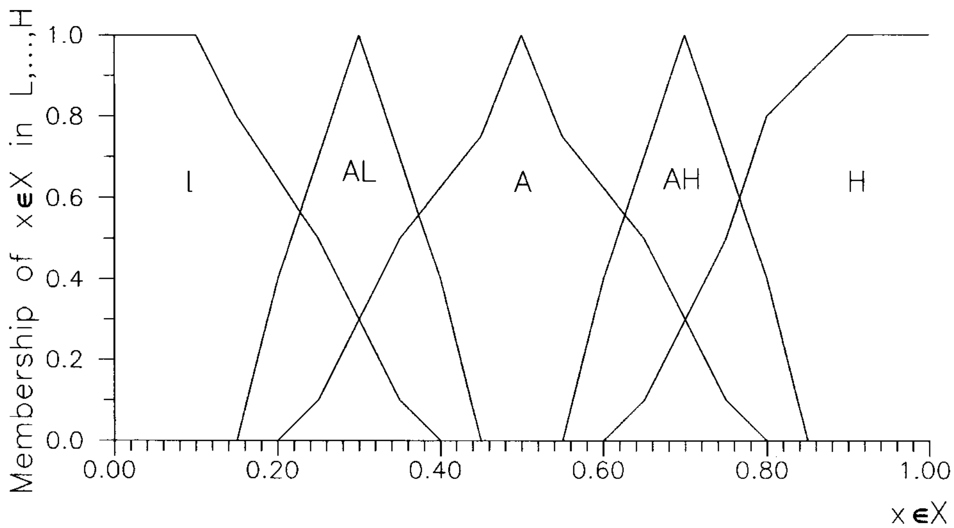

Fig. 1. Membership functions of linguistic values L = low, AL = about low, A = average, AH = about high, and H = high.

Rule of Inference (CRI) proposed by Zadeh [27], is based on the composition of fuzzy relations. An example of fuzzy implication comes next.

Let A, A^* be fuzzy sets on X, B, B^* be fuzzy sets on Y, and the statements

Premise:	X is A^*	(Observation)
Implication:	If X is A then Y is B	(Expert Rule)
Conclusion:	Y is B^*	(Consequence).

Note that A and A^* are simply different fuzzy sets of the same universal set; the same is true for B and B^*. In the above inference schema, an observation is combined with an expert rule providing the consequence, which in turn is the advice to the decision maker. The conclusion B^* can be obtained from the CRI by taking the composition of the observation A^* and the fuzzy conditional $A \rightarrow B$ as follows:

$$B^* = A^{*\circ}(A \rightarrow B)$$

where "$\circ$" is the max-min composition. In the membership function domain, the CRI is

$$\mu_{B^*}(y_j) = \bigvee_i [\mu_{A^*}(x_i) \wedge \mu_{A\rightarrow B}(x_i, y_j)], \quad i = 1, 2, \cdots, I \quad j = 1, 2, \cdots, J$$

where $\mu_{B^*}(y_j)$ is the membership grade of the jth element of $B^*, \mu_{A^*}(xi)$ is the membership grade of the ith element of $A^*, \mu_{A\rightarrow B}(x_i, y_j)$ is the membership grade of the ijth element of the implication relation $A \rightarrow B$, and $\vee, \wedge$ denote maximum and minimum, respectively.

The CRI, based on the max-min-standard complement, is a method suggested by Zadeh [27] and is a commonly used approximate reasoning schema. Later research [28], [29] revealed that for desirable inference, there has to be an appropriate match between the conjunction and implication operators that define the composition "$\circ$" operator in $B^* = A^{*\circ}(A \rightarrow B)$. According to these results, inference based on the max-min-standard complement does not produce a desirable inference. In real-life applications, however, it is suggested that an appropriate conjunction and implication operator combination should be chosen for a desirable inference. Having noted this, we will continue our presentation in max-min schema to provide a prototype framework for the potential application of fuzzy logic in the determination of machine flexibility.

A. Fuzzy Interval Implication

We define three input linguistic variables for each machine, namely, *setup (S), versatility (V),* and *adjustability (R),* which take on linguistic values: *low (L), about low (AL), average (A), about high (AH),* and *high (H).* The various setup, versatility, and adjustability values, which are denoted by $s \in S, v \in V$, and $r \in R$, respectively, construct the base variable values within a context that is defined as $x \in X$. Some representative membership grades $\mu(x)/x$ of the linguistic values for each of the above variables are given in Table I, while the corresponding membership function curves are shown in Fig. 1.

It should be noted that the linguistic values shown in Fig. 1 are commonly used by all variables but they are scaled into the interval [0, 1]. The physical domain of the linguistic variables is defined by the available technologies. For example, setup times s_i for mc's range from zero to six hours; or, equivalently, the physical domain of S is [0, 6]. v_i range from zero to 35 operations over a set of products, and r_i range from zero to 1.5 cubic meters.

In practice, in addition to the fuzziness of various concepts, we encounter fuzziness in the way these concepts are related to each other. For example, managers may not have a precise idea not only of how to define versatility and adjustability but also how these parameters should be logically combined to obtain a flexibility assessment. Therefore, their knowledge can be represented in the form of "if $\langle antecedents \rangle$ then $\langle consequent \rangle$" rules, where the implication operator and the connectives among antecedents are fuzzy. These rules include statements that are close to natural language and can be extracted via knowledge-acquisition methodologies [30], such as interviews or questionnaires. For a detailed exposition in knowledge-acquisition studies in the context of flexibility, see [31] and [32]. The linguistic rule base of our study contains

TABLE I
MEMBERSHIP GRADES OF THE LINGUISTIC VALUES

Low	(1/.1, .8/.15, .5/.25, .1/.35, 0/.45, 0/.5, 0/.65, 0/.65, 0/.75, 0/.85, 0/.9)
About Low	(0/.1, 0/.15, .4/.25, 1/.35, .4/.45, 0/.5, 0/.65, 0/.75, 0/.85, 0/.9)
Average	(0/.1, 0/.15, .1/.25, .5/.35, .75/.45, 1/.5, .5/.65, .1/.75, 0/.85, 0/.9)
About High	(0/.1, 0/.15, 0/.25, 0/.35, 0/.45, 0/.5, .4/.65, 1/.75, .4/.85, 0/.9)
High	(0/.1, 0/.15, 0/.25, 0/.35, 0/.45, 0/.5, .1/.65, .5/.75, .8/.85, 1/.9)

$5^3 = 125$ rules, which include all variations of the linguistic values, i.e., five linguistic values for each of the three linguistic variables. The rules, which represent the expert knowledge on how the variables affect flexibility, are of the following form:

IF	setup	is	$s \in S$
AND	versatility	is	$v \in V$
AND	adjustability	is	$r \in R$
THEN	machine flexibility	is	$mf \in MF.$

Let $T = \{L, AL, A, AH, H\}$ be the set of linguistic values for all the four variables $S, V, R,$ and MF, and let $T_S, T_V, T_R,$ and $T_{\text{MF}} \in T$ be the linguistic value sets for $S, V, R,$ and MF, respectively. Then the above rule can be written compactly as follows:

$$\textbf{IF } S \text{ is } T_S \text{ AND } V \text{ is } T_V \text{ AND } R \text{ is } T_R \textbf{ THEN } MF \text{ is } T_{\text{MF}} \tag{2}$$

where $T_S, T_V, T_R,$ and $T_{\text{MF}} \in T$. A graphical illustration of a subset of the rule base is shown in Fig. 2.

The linguistic values of the variables setup, versatility, and adjustability that represent the inexact knowledge of the experts are fuzzy sets. More important, logical AND connectives are also fuzzy, as we already mentioned. If the AND operator were crisp, as in two-valued logic, the result of the statement "A AND B" would be unambiguous. Here, however, we are faced with the problem of expressing such ambiguities. There are many ways to express fuzziness in an AND operator [20]. In this paper, we chose the fuzzy interval representation of connectives [33] because this mechanism provides a reasonable interval of uncertainty for each operator in the sense of experimental [20] and theoretical results [33], [36].

In this section, MF is the conclusion of an approximate reasoning method, which can be schematically described by the following:

Observation:	S is T_S^* AND V is T_V^* AND R is T_R^* (Facts)
Linguistic Rule:	**IF** S is T_S AND V is T_V AND R is T_R **THEN** MF is T_{MF} (Knowledge)
Conclusion:	MF is T_{MF}^* (Measurement)

where $T_S^*, T_V^*, T_R^*,$ and $T_{\text{MF}}^* \in T^*$ are linguistic value sets in general different than $T_S, T_V, T_R,$ and $T_{\text{MF}} \in T$. Informally, approximate reasoning is the process by which, given an observation, a conclusion is deduced from a collection of fuzzy rules that describe the relationship among the system variables. Here, these relationships are quantified by utilizing the interval-valued representation of AND and IF-THEN connectives. This leads to the construction of lower and upper bounds on the nonspecificity of a combination of linguistic concepts based on disjunctive and conjunctive normal forms of the linguistic values [33], [36]. For example, for the AND operator that connects two fuzzy sets A and B, we now obtain a lower and an upper bound. The rationale is that by defining a bounded region for the membership function of "A AND B," we capture the dispersion of imprecise knowledge, i.e., its nonspecificity. In the appendix, we provide definitions of the interval-valued representation of connectives as well as other related background material. Generally, if LC(•) is a logical connective for (•), then [33]

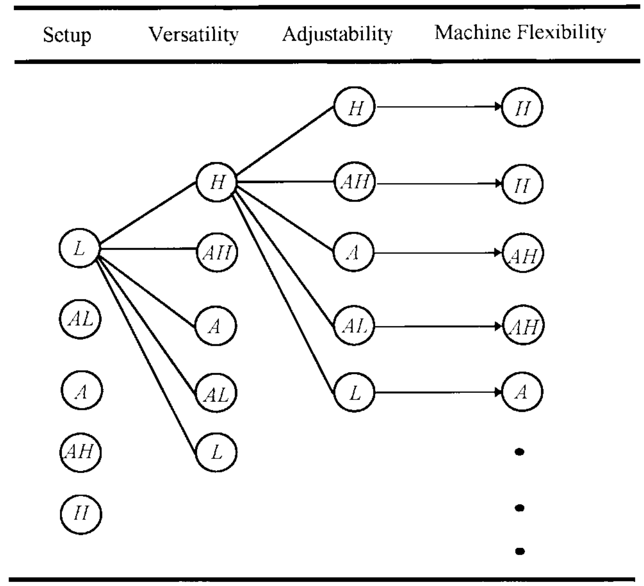

Fig. 2. A subset of the linguistic rule base.

$$\text{DNF}(\bullet) \subseteq \text{LC}(\bullet) \subseteq \text{CNF}(\bullet) \tag{3}$$

where DNF stands for "disjunctive normal form" and CNF stands for "conjunctive normal form." Or, equivalently, in the membership function domain

$$r_{\text{DNF}}(\bullet) \leq r_{\text{LC}}(\bullet) \leq r_{\text{CNP}}(\bullet).$$

For the statement "A AND B," where A and B are assumed to be fuzzy sets, the lower bound $L_{\text{AND}}{}^{[2]}$ is [33]–[35]

$$L_{\text{AND}}{}^{[2]} = \text{DNF}(A \text{ AND } B) \tag{4}$$

and the corresponding upper bound $U_{\text{AND}}{}^{[2]}$ is

$$U_{\text{AND}}{}^{[2]} = \text{CNF}(A \text{ AND } B). \tag{5}$$

From (3), we obtain

$$L_{\text{AND}}{}^{[2]} = \text{DNF}(A \text{ AND } B) \subseteq (A \text{ AND } B) \subseteq \text{CNF}(A \text{ AND } B) = U_{\text{AND}}{}^{[2]}.$$

Following (4) for the statement "A AND B AND C"

$$L_{\text{AND}}{}^{[3]} = \text{DNF}\left(L_{\text{AND}}{}^{[2]} \text{ AND } C\right) \tag{6}$$

while the upper bound (see also the appendix) is

$$U_{\rm AND}{}^{[3]} = \Big[{\rm CNF}\Big(L_{\rm AND}{}^{[2]} \text{ AND } C\Big)\Big] \cup \Big[{\rm CNF}\Big(U_{\rm AND}{}^{[2]} \text{ AND } C\Big)\Big]. \quad (7)$$

In the same manner, for the statement "IF A THEN B," the bounds for the IF-THEN $(\rightarrow)$ connective are $L_{(\rightarrow)} =$ DNF$(A \rightarrow B)$ and $U_{(\rightarrow)} =$ CNF$(A \rightarrow B)$. From (3), we have

$$L_{(\rightarrow)} = {\rm DNF}(A \rightarrow B) \subseteq (A \rightarrow B) \subseteq {\rm CNF}(A \rightarrow B) = U_{(\rightarrow)}.$$

All the above expressions are written in the fuzzy-set domain. They have their equivalents in the membership function domain [33], [36]. In the appendix, we exhibit the mathematical meaning of the fuzzy rules within an approximate reasoning procedure.

As previously stated, the rule base contains 125 three-antecedent rules such as

IF S is L AND V is L AND R is A **THEN** MF is A.

These rules represent the core of knowledge needed for the approximate reasoning process. The meaning of the above rule is that, according to the experts, the flexibility of a machine attains an (A)verage value when the antecedents assume the values (L)ow, (L)ow, and (A)verage. The knowledge of the experts about flexibility in the consequent side of the rule is imprecise. We use the fuzzy interval implication to handle this imprecision by capturing the scattering of knowledge around a central tendency. It should be emphasized, however, that the selection of the interval-valued representation of connectives is not restrictive. One can use other operators to achieve a desirable performance of the reasoning procedure.

For a given input $T_S, T_V, T_R \in T$, we first compute the bounds of the AND connective in the fuzzy-set domain. From (6), we obtain

$$L_{\rm AND}{}^{[3]} = {\rm DNF}(T_S \text{ AND } T_V \text{ AND } T_R) = T_S \cap T_V \cap T_R. \quad (8)$$

Similarly, from (7)

$$\begin{aligned} U_{\rm AND}{}^{[3]} = \cup \{&{\rm CNF}[(T_S \cap T_V) \text{ AND } T_R], \\ &{\rm CNF}[(T_S \cup T_V) \cap (T_S \cup T_V^c) \cap (T_S^c \cup T_V) \text{ AND } T_R]\} \\ = \cup \{&[(T_S \cap T_V) \cup T_R] \cap [(T_S \cap T_V) \cup T_R^c] \\ &\cap [(T_S \cap T_V)^c \cup T_R], \\ &[(T_S \cup T_V) \cap (T_S \cup T_V^c) \cap (T_S^c \cup T_V)] \cup T_R \\ &\cap [(T_S \cup T_V) \cap (T_S \cup T_V^c) \cap (T_S^c \cup T_V)] \\ &\cup T_R^c \cap [(T_S \cup T_V) \cap (T_S \cup T_V^c) \\ &\cap (T_S^c \cup T_V)]^c \cup T_R\}. \end{aligned} \quad (9)$$

To simplify the implication, we compute the center $C_{\rm AND}$ of the region determined by (8) and (9)

$$C_{\rm AND} = \tfrac{1}{2}\Big[L_{\rm AND}{}^{[3]} + U_{\rm AND}{}^{[3]}\Big]. \quad (10)$$

This is a heuristic simplification to control the dispersion of uncertainty from one computation to the next. The bounds for the implication $(C_{\rm AND} \rightarrow T_{\rm MF})$ are given in the set domain as follows:

$$\begin{aligned} L_{(\rightarrow)} &= {\rm DNF}(C_{\rm AND} \rightarrow T_{\rm MF}) \\ &= (C_{\rm AND} \cap T_{\rm MF}) \cup (C_{\rm AND}^c \cap T_{\rm MF}) \\ &\quad \cup (C_{\rm AND}^c \cap T_{\rm MF}^c) \end{aligned} \quad (11)$$

$$U_{(\rightarrow)} = {\rm CNF}(C_{\rm AND} \rightarrow T_{\rm MF}) = C_{\rm AND}^c \cup T_{\rm MF}. \quad (12)$$

The center of the implication is

$$C_{(\rightarrow)} = \tfrac{1}{2}[L_{(\rightarrow)} + U_{(\rightarrow)}] \quad (13)$$

Equation (13) again is a heuristic simplification that computes the center of the region in which flexibility potentially belongs by taking into account the joint interaction of the observations about MF through the AND and implication operators.

Suppose now that instead of the five linguistic values L, AL, A, AH, and H, a user defines a finer gradation of the variables of interest by introducing a value between L and AL. Then the new inputs would be T_S^*, T_V^*, and $T_R^* \in T^*$, and consequently, the center of the AND operator would yield $C_{\rm AND}^* \neq C_{\rm AND}$. The value of flexibility $T_{\rm MF}^*$ is then given by the CRI

$$T_{\rm MF}^* = C_{\rm AND}^* \circ C_{(\rightarrow)} \quad (14)$$

where "$\circ$" represents the max-min composition. By interpreting (14) in the computational membership domain, we derive the membership function of $T_{\rm MF}^*$ as follows:

$$\mu_{T_{\rm MF}^*}(y_j) = \bigvee_i [\mu_{C_{\rm AND}^*}(x_i) \wedge \mu_{C_{(\rightarrow)}}(x_i, y_i)], \quad i = 1, 2, \cdots, I, \quad j = 1, 2, \cdots, J \quad (15)$$

where $\mu_{T_{\rm MF}^*}(y_j)$ is the membership function value of the jth element of $T_{\rm MF}^*$, i.e., the inferred membership value of the element of $y_j, \mu_{C_{\rm AND}^*}$ is the membership function value of the ith element of $C_{\rm AND}^*$, i.e., the observed membership value of x_i, and $\mu_{C_{(\rightarrow)}}(x_i, y_i)$ is the membership function value of the ijth element of the implication relation matrix.

B. An Example

Suppose that for a particular machine, we have the following inputs (observations): 1) setup time is about low, 2) versatility is high, and 3) adjustability is average. The rule whose antecedents match the observations and contain the desirable information about the value of MF is

IF S is AL AND V is H AND R is A **THEN** MF is AH

or, equivalently, AL AND H AND $A \rightarrow AH$.

In calculations, we use the membership grades of the linguistic values that are shown in Table I. Following the procedure described above, and expressing operators in the membership domain, we first compute the bounds of the AND operator. Max-min operators are employed to carry out the computations, and from (8), replacing $\cap$ with $\wedge$, we have

$$\begin{aligned} L_{\text{AND}}^{[3]} &= AL \wedge H \wedge A \\ &= (0/.1, 0/.15, .3/.25, 1/.35, .3/.45, 0/.5, \\ &\quad 0/.65, 0/.75, 0/.85, 0/.9) \\ &\quad \wedge (0/.1, 0/.15, 0/.25, 0/.35, 0/.45, 0/.5, \\ &\quad\quad .1/.65, .5/.75, .8/.85, 1/.9) \\ &\quad \wedge (0/.1, 0/.15, .1/.25, .5/.35, .75/.45, 1/.5, \\ &\quad\quad .5/.65, .1/.75, 0/.85, 0/.9) \\ &= (0/x), \qquad \text{where } x \in [0, 1]. \end{aligned}$$

The upper bound is computed from (9), where union and intersection correspond to max ($\vee$) and min ($\wedge$) operators, respectively. Thus

$$\begin{aligned} U_{\text{AND}}^{[3]} = (&.3/.25, .5/.35, .3/.45, .3/.5, .5/.65, \\ &.5/.75, .2/.85) \end{aligned}$$

and the center of the AND operator is

$$\begin{aligned} C_{\text{AND}} = (&.15/.25, .25/.35, .15/.45, .15/.5, .25/.65, \\ &.25/.75, .1/.85). \end{aligned}$$

A sufficient condition for meaningful implication [34] is that C_{AND} should be normalized (i.e., $\exists x, : r(x) = 1$). Dividing C_{AND} by 0.25, which is the maximum membership grade, we obtain

$$C_{\text{AND}} = (.6/.25, 1/.35, .6/.45, .6/.5, 1/.65, 1/.75, .4/.85).$$

Now we apply the implication operator to extend the range of experts' advice about flexibility. The bounds of implication involve the normalized result of the logical connection (C_{AND}) and the knowledge provided by experts (T_{MF}). These bounds, according to (11) and (12), are

$$\begin{aligned} L_{(\rightarrow)} &= (C_{\text{AND}} \wedge AH) \vee (C^c_{\text{AND}} \wedge AH) \vee (C^c_{\text{AND}} \wedge AH^c) \\ U_{(\rightarrow)} &= C^c_{\text{AND}} \vee AH \end{aligned}$$

and $C_{\text{AND}} = (.6/.25, 1/.35, .6/.45, .6/.5, 1/.65, 1/.75, .4/.85)$, $AH = (.4/.65, 1/.75, .4/.85)$. All possible values of the bounds $L_{(\rightarrow)}$ and $U_{(\rightarrow)}$ in the membership domain will be arranged in two 7 × 3 matrices, where the number of rows equals the number of values for C_{AND} (0.25, 0.35, 0.45, 0.5, 0.65, 0.75, 0.85) and the number of columns equals the number of values of AH (0.65, 0.75, 0.85). Thus

$$L_{(\rightarrow)} = \begin{bmatrix} .4 & .6 & .4 \\ .4 & 1 & .4 \\ .4 & .6 & .4 \\ .4 & .6 & .4 \\ .4 & 1 & .4 \\ .4 & 1 & .4 \\ .6 & .6 & .6 \end{bmatrix}, \qquad U_{(\rightarrow)} = \begin{bmatrix} .4 & 1 & .4 \\ .4 & 1 & .4 \\ .4 & 1 & .4 \\ .4 & 1 & .4 \\ .4 & 1 & .4 \\ .4 & 1 & .4 \\ .6 & 1 & .6 \end{bmatrix}$$

where, for example, $1_{11} = .4 = (.6 \wedge .4) \vee (.4 \wedge .4) \vee (.4 \wedge .6)$ and $u_{11} = .4 = (.4 \vee .4)$. From (13) and for the machine under study, we have

$$C_{(\rightarrow)} = \begin{bmatrix} .4 & .8 & .4 \\ .4 & 1 & .4 \\ .4 & .8 & .4 \\ .4 & .8 & .4 \\ .4 & 1 & .4 \\ .4 & 1 & .4 \\ .6 & .8 & .6 \end{bmatrix}.$$

Now consider the CRI. By using the above matrix and the center of the AND operator, we obtain the membership function of machine flexibility. For example, suppose that we have a slightly different input with $C^*_{\text{AND}} = (.5/.25, .9/.35, .6/.45, .6/.5, 1/.65, .8/.75, .6/.85)$. The membership function of flexibility is

$$\begin{aligned} T'_{\text{MF}} = C^*_{\text{AND}} {}^\circ C_{(\rightarrow)} &= [.5\ .9\ .6\ .6\ 1\ .8\ .6]^\circ \begin{bmatrix} .4 & .8 & .4 \\ .4 & 1 & .4 \\ .4 & .8 & .4 \\ .4 & .8 & .4 \\ .4 & 1 & .4 \\ .4 & 1 & .4 \\ .6 & .8 & .6 \end{bmatrix} \\ &= (.6/.65\ 1/.75\ .6/.85) \end{aligned}$$

where [from (15)]

$$\begin{aligned} \mu_{T^*_{\text{MF}}}(.65) = \mu_{T^*_{\text{MF}}}(.85) &= (.5 \wedge .4) \vee (.9 \wedge .4) \vee (.6 \wedge .4) \vee (.6 \wedge .4) \\ &\quad \vee (1 \wedge .4) \vee (.8 \wedge .4) \vee (.6 \wedge .6) = .6 \end{aligned}$$

and

$$\begin{aligned} \mu_{T^*_{\text{MF}}}(.75) = &(.5 \wedge .8) \vee (.9 \wedge 1) \vee (.6 \wedge .8) \vee (.6 \wedge .8) \\ &\vee (1 \wedge 1) \vee (.8 \wedge 1) \vee (.6 \wedge .8) = 1. \end{aligned}$$

The methodology above is useful in assessing individual machine flexibilities, and thus it is an important tool to a manager in choosing a specific manufacturing configuration. In practice, however, managers would prefer a single number over membership functions to obtain a better feeling of flexibility. To convert the membership function of flexibility into a single point-wise value, we use a procedure called *defuzzification,* which is widely used in the area of fuzzy control. Among various defuzzification methods, we choose the so-called center-of-gravity formula, which is the most frequently referenced in the literature. Then the crisp value of machine flexibility is given by

$$\text{def } T^*_{\text{MF}} = \frac{\sum_j y_j \cdot \mu_{T^*_{\text{MF}}}(y_j)}{\sum_j \mu_{T^*_{\text{MF}}}(y_j)}, \qquad j = 1, 2, \cdots, J \qquad (16)$$

where $\mu_{T^*_{\text{MF}}}(y_j)$ is the membership grade of point y_j. Applying (16) on the membership function of T^*_{MF} given by the above example, we derive

$$\text{def } T^*_{\text{MF}} = \frac{0.65 \cdot 0.6 + 0.75 \cdot 1 + 0.85 \cdot 0.6}{0.6 + 1 + 0.6} = 0.75.$$

IV. Conclusion

We have presented two measurement methodologies for the assessment of machine flexibility, which incorporate three major operational aspects: setup, versatility, and adjustability. The first measure provides a relative evaluation, as it is based on comparisons among crisp machine characteristics. This measure places emphasis on beliefs of the managers as to the importance of each operational characteristic in the measurement of flexibility and results in a situation-specific measurement. In the second measure, the value of flexibility is deduced from a fuzzy reasoning process in the context of a rule-based system. No analytic formulas or numerical data, which are difficult to find in practice, are required in order to assess flexibility. The main characteristic of this approach is that it utilizes natural language expressions and thus captures the knowledge about the measurement of machine flexibility. Furthermore, it should be noted that the fuzzy measure is proposed from a structural perspective, as one can use linguistic rules of different form or other approximate reasoning procedures to achieve a desirable performance in a given context. The measurement schemes proposed in this paper appear to have two advantages.

1) They are easy to use and interpret and suitable in comparing alternative machine designs. Equation (1) allows flexibility comparisons between alternate machines provided precise numerical data exist. In practice, however, the fuzzy measurement scheme looks more attractive, as it utilizes already gained knowledge together with natural language ratings, which are favored by managers.
2) They combine three different parameters of a production system, which play an important role in defining and measuring machine-level flexibility.

A topic of future research is the development of a manufacturing system measure able to capture the uncertainty in the interrelationships among the various types of flexibility. To accomplish that, further research is needed to define measures for other flexibility types such as routing, material handling, process, and labor using fuzzy-logic methodologies.

Appendix

A. Definitions and Background

Definition 1: Two fuzzy sets A and B are equal $(A = B)$ if and only if

$$\forall\, x \in X\colon r_A(x) = r_B(x).$$

Definition 2: A is a subset of $B (A \subseteq B)$ if and only if

$$\forall\, x \in X\colon r_A(x) \le r_B(x).$$

Now, for $r_A(x) = a$ and $r_B(x) = b$, we have Definition 3.

Definition 3: A triangular norm or t-norm T denotes a class of binary functions, which satisfies the following conditions.

Boundary conditions:	$T(0,a) = 0, T(a,1) = T(1,a) = a$.
Monotonicity:	$T(a,b) \le T(c,d)$ whenever $a \le c$, $b \le d$.
Symmetry:	$T(a,b) = T(b,a)$.
Associativity:	$T(T(a,b),c) = T(a,T(b,c))$.

Examples of t-norms T are min $(a,b), ab$, max $(0, a+b-1)$, etc. The same conditions hold for the t-conorm S. Every t-norm T determines a unique dual t-conorm S, which is defined by

$$S(a,b) = 1 - T(1-a, 1-b)$$

or

$$T(a,b) = 1 - S(1-a, 1-b).$$

Examples of t-conorms S are max $(a,b), a+b-ab$ and min $(1, a+b)$. From [26], [37], and other investigations, it can be argued that the t-norms-conorms are suitable candidates for conjunctions and disjunctions in many-valued logic.

B. Interval-Valued Fuzzy Sets [33]–[35]

Every linguistic proposition can be represented by an interval-valued fuzzy set. Generally, if $\mathbf{LC}(\bullet)$ is a logical connective for $(\bullet)$, then

$$\text{DNF}(\bullet) \subseteq \text{LC}(\bullet) \subseteq \text{CNF}(\bullet).$$

For the propositions AND, OR, and IF ... THEN, LC$(\bullet)$ is defined as follows:

$$\begin{aligned} \text{DNF}(A \text{ AND } B) &= A \cap B \subseteq \mathbf{LC}(\boldsymbol{A} \textbf{ AND } \boldsymbol{B}) \\ &\subseteq (A \cup B) \cap (A \cup B^c) \cap (A^c \cup B) \\ &= \text{CNF}(A \text{ AND } B) \qquad (17) \\ \text{DNF}(A \text{ OR } B) &= (A \cap B) \cup (A \cap B^c) \cup (A^c \cap B) \\ &\subseteq \mathbf{LC}(\boldsymbol{A} \textbf{ OR } \boldsymbol{B}) \subseteq A \cup B \\ &= \text{CNF}(A \text{ OR } B) \qquad (18) \\ \text{DNF}(A \to B) &= (A \cap B) \cup (A^c \cap B) \cup (A^c \cap B^c) \\ &\subseteq \mathbf{LC}(\boldsymbol{A} \to \boldsymbol{B}) \subseteq A^c \cup B \\ &= \text{CNF}(A \to B). \qquad (19) \end{aligned}$$

Equations (17)–(19) are written in the fuzzy-set domain, and consequently, $\cap, \cup$, and ^{c} are the well-known intersection, union, and complement operators, respectively, which correspond to appropriate t-norm T, t-conorm S, and complement ^{c} operators in the domain of membership functions, respectively. Thus, the interval-valued fuzzy set in the membership domain is defined as follows:

$$\mu_{\text{DNF}(S,T,^c)} \le \mu_{\text{LC}} \le \mu_{\text{CNF}(S,T,^c)}$$

where $T, S,$ and ^{c} are the t-norm, t-conorm, and complement operators, respectively.

1) Three-Antecedent Intersection [35]: Consider a three-antecedent rule as follows:

IF X_1 is A_1 AND X_2 is A_2 AND X_3 is A_3 THEN Y is B

or

$$A_1 \text{ AND } A_2 \text{ AND } A_3 \rightarrow B.$$

It is known that for a two-antecedent intersection, we have

$$\begin{aligned} L^{[2]} &= \text{DNF}(A_1 \text{ AND } A_2) \subseteq \mathbf{LC}^{[2]}(\text{AND}) \\ &\subseteq \text{CNF}(A_1 \text{ AND } A_2) = U^{[2]}. \end{aligned} \tag{20}$$

For the three-antecedent intersection A_1 AND A_2 AND A_3, and according to (17) and (20), we have two families of interval-valued fuzzy sets and, as follows:

$$\begin{aligned} \text{DNF}(L^{[2]} \text{ AND } A_3) &\subseteq \mathbf{LC}^{[3]}(\text{AND}) \\ &\subseteq \text{CNF}(L^{[2]} \text{ AND } A_3) \end{aligned} \tag{21}$$

$$\begin{aligned} \text{DNF}(U^{[2]} \text{ AND } A_3) &\subseteq \mathbf{LC}^{[3]}(\text{AND}) \\ &\subseteq \text{CNF}(U^{[2]} \text{ AND } A_3). \end{aligned} \tag{22}$$

In general, it is not known whether CNF($U^{[2]}$ AND A_3) is larger or smaller than CNF($L^{[2]}$ AND A_3), since these expressions are nonmonotonic.

Theorem [35]: The lower bound of the three-antecedent intersection "A_1 AND A_2 AND A_3" is DNF($L^{[2]}$ AND A_3), and its upper bound is $\cup\{$CNF($L^{[2]}$ AND A_3), CNF($U^{[2]}$ AND A_3)$\}$, where $\cup$ is set union, which corresponds to an appropriate t-conorm S in the membership domain.

Proof: Let $L^{[2]}, U^{[2]}$, and A_3 be fuzzy sets with membership functions values l, u, and $a \in [0, 1]$, respectively, and $L^{[2]} \subseteq U^{[2]}$, or in the membership domain $\mu_{L^{[2]}} = l \leq u = \mu_{U^{[2]}}$. It is clear that $u^c \leq l^c$, where $u^c = 1 - u$ and $l^c = 1 - l$. For the lower bound, we notice that $L^{[2]} \subseteq U^{[2]}$ implies

$$\begin{aligned} \text{DNF}\left(L^{[2]} \text{ AND } A_3\right) &= \left(L^{[2]} \cap A_3\right) \subseteq \left(U^{[2]} \cap A_3\right) \\ &= \text{DNF}\left(U^{[2]} \text{ AND } A_3\right) \end{aligned}$$

and the result follows directly, as DNF$\left(L^{[2]}\text{ AND } A_3\right)$ is the lowest of the lower bounds in (21) and (22). For the upper bound, we observe that there is no explicit relationship among CNF$\left(L^{[2]}\text{ AND } A_3\right)$ and CNF$\left(U^{[2]}\text{ AND } A_3\right)$. In particular, from (17), we have

$$\begin{aligned} \text{CNF}\left(L^{[2]} \text{ AND } A_3\right) &= \left(L^{[2]} \cup A_3\right) \cap \left(L^{[2]} \cup A_3^c\right) \\ &\quad \cap \left(L^{[2]c} \cup A_3\right) \\ \text{CNF}\left(U^{[2]} \text{ AND } A_3\right) &= \left(U^{[2]} \cup A_3\right) \cap \left(U^{[2]} \cup A_3^c\right) \\ &\quad \cap \left(U^{[2]c} \cup A_3\right) \end{aligned}$$

or, equivalently, in the membership domain

$$\mu_{\text{CNF}(L^{[2]} \text{ AND } A_3)} = T(T(S(l,a), S(l,a^c)), S(l^c,a)) \tag{23}$$

$$\mu_{\text{CNF}(U^{[2]} \text{ AND } A_3)} = T(T(S(u,a), S(u,a^c)), S(u^c,a)). \tag{24}$$

From the monotonicity condition, we know that $S(l,a) \leq S(u,a), S(l,a^c) \leq S(u,a^c)$, and

$$T(S(l,a), S(l,a^c)) \leq T(S(u,a), S(u,a^c))$$

but $S(u^c,a) \leq S(l^c,a)$, and thus depending on the relations among l, u, and a, $\mu_{\text{CNF}(L^{[2]} \text{ AND } A_3)}$ could be either larger or smaller than $\mu_{\text{CNF}(U^{[2]} \text{ AND } A_3)}$, i.e., CNF($A$ AND B) is nonmonotonic. Consequently, the upper bound $U^{[3]}$ of the upper bounds of (20) and (21) is given in the fuzzy-set domain by the union of CNF($L^{[2]}$ AND A_3) and CNF($U^{[2]}$ AND A_3), i.e.,

$$U^{[3]} = \cup\left\{\text{CNF}\left(L^{[2]} \text{ AND } A_3\right), \text{CNF}\left(U^{[2]} \text{ AND } A_3\right)\right\}. \tag{25}$$

■

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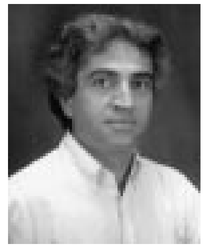

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